Discrete Outcome Quantum Sensor Networks

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Introduction

Quantum sensors, being strongly sensitive to external disturbances, can measure various physical phenomena with extreme sensitivity. While the hardware instability is a liability for quantum computing, it is an asset for quantum sensing.

Known: A classical sensor network is a network of spatially dispersed sensors that monitor physical conditions of the environment such as temperature, sound, pollution levels, humidity, wind and wireless spectrum.
Question: What is a quantum sensor network?
Answer: A network of spatially dispersed sensors that leverage the quantum properties of light and matter, e.g., quantum object, coherence, and entanglement.

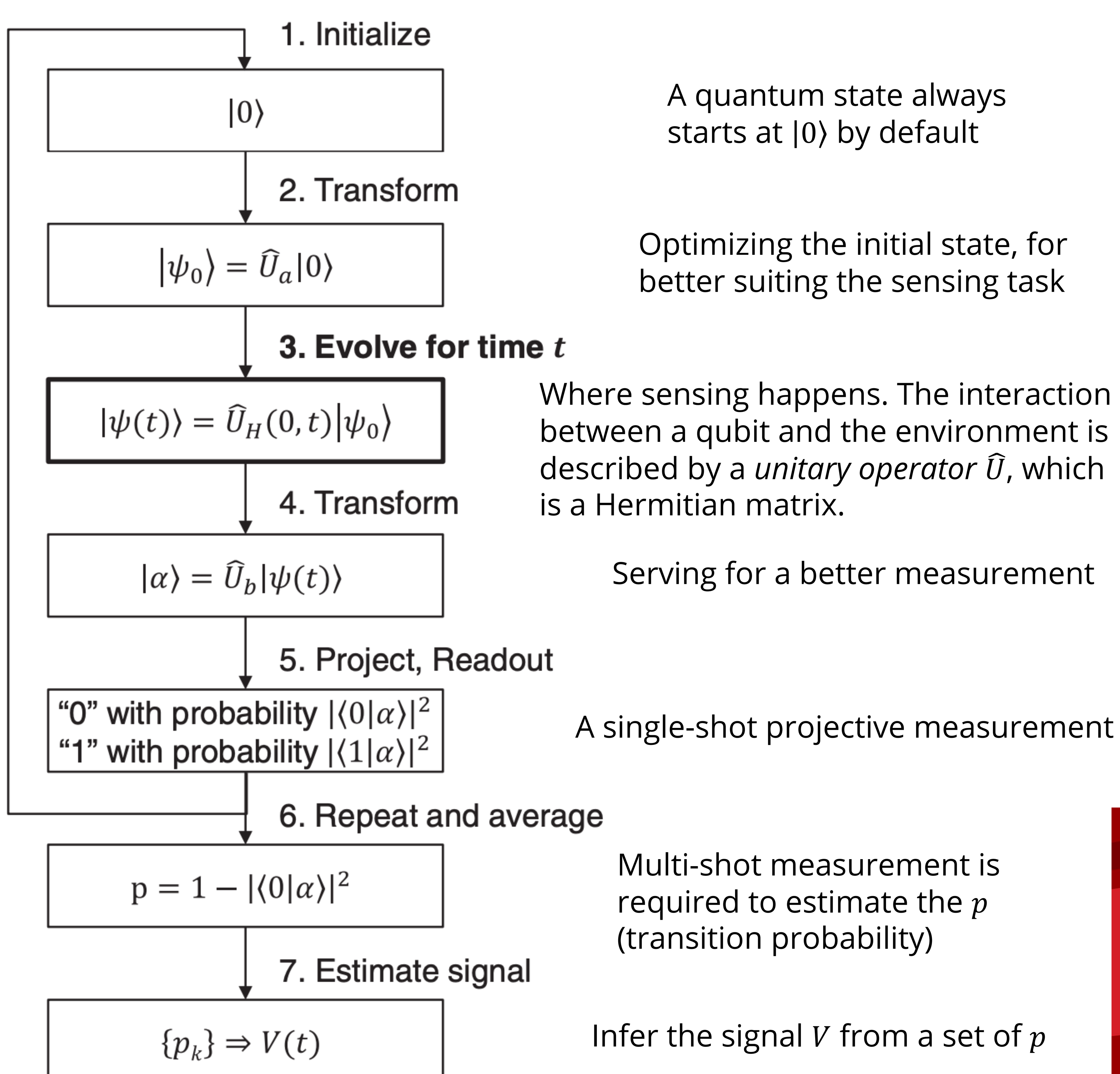

Fig. 1. A quantum sensing protocol summarized in 7 steps. Here the measurement is projective. A more general measurement is POVM.

Problem Definition

We have N quantum sensors, each of which is a represented by a qubit, initially in a N-qubit state $|\psi\rangle$. One of the detectors has its state altered by an interaction with the environment, described by a unitary operator U, and we would like to know which. This leads to a quantum state discrimination problem. We are discriminating the N quantum states: $|\phi_i\rangle = (I^{\otimes(i-1)} \otimes U \otimes I^{\otimes(n-i)})\,|\psi\rangle$, $i = 1 \cdots N$
E.g., when N=2, discriminate between $(U \otimes I)|\psi\rangle$ and $(I \otimes U)|\psi\rangle$

This leads to two questions. How should we choose the initial state $|\psi\rangle$ (step 1&2 in Fig. 1) and how should we choose a measurement (step 4&5 in Fig. 1) to accomplish this optimally? This is essentially a double optimization problem. For the choosing a measurement, we resort to a numerical method based on semi-definite programming. **Choosing the initial state $|\psi\rangle$** is the focus of this poster. Theorem 1 and theorem 2 are both theorems about the initial state.

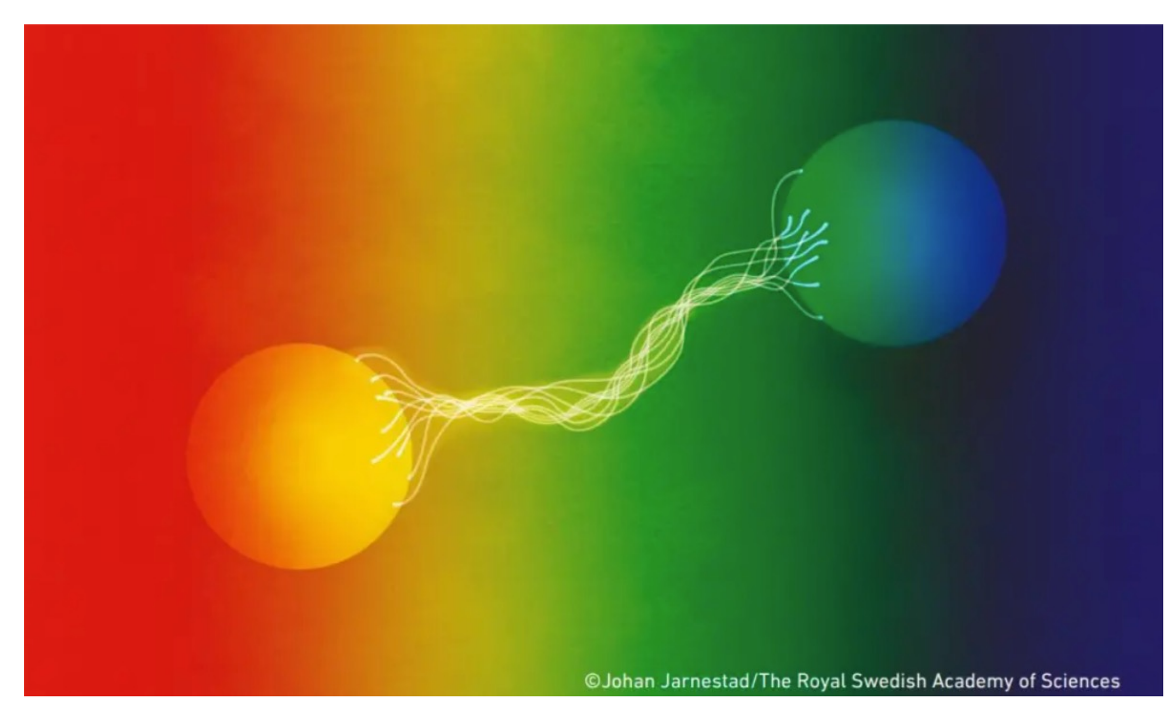

Fig. 2. Illustration of quantum entanglement. The 2022 Nobel prize in physics is awarded to three pioneers in quantum entanglement. Entanglement is an important resource in many applications of quantum sensing and quantum communication.

KEY FINDINGS

→ **An entangled initial state can improve the probability of detecting whether a qubit detector is interacting with the environment or not.**

→ **Theorem 1. Under our problem, gives the conditions when the quantum states can be mutually orthogonal.**

→ **Theorem 2. Gives the expression of the optimal initial quantum state under the condition specified by Theorem 1.**

Theorems 1 & 2

Theorem 1: Let U be a unitary operator. Without loss of generality, let the eigenvectors of U be $U\,|u_\pm\rangle = e^{\pm i\theta}\,|u_\pm\rangle$. Let $|\psi\rangle$ denote the initial (possibly, entangled) state of the n sensors. Let $|\phi_i\rangle = (I^{\otimes(i-1)} \otimes U \otimes I^{\otimes(n-i)})\,|\psi\rangle$, where U appears in the i^{th} position. For any $\theta \in [T, 180-T]$ degrees, there exists a $|\psi\rangle$ such that the set of n states $\{|\phi_i\rangle\}$ are mutually orthogonal, where T is given by:

$$T = \frac{1}{2}\arccos\left[-\left(min_{1\le k\le(n-1)}\frac{\binom{(k)}{(2)}+\binom{(n-k)}{(2)}}{\binom{(k)}{(1)}\times\binom{(n-k)}{(1)}}\right)\right]$$

The values of T for increasing n are: 60 ($n=4$), 65.9 ($n=5,6$), 69.3 ($n=7,8$), 71.6 ($n=9,10$).

We claim that the converse of the above is also true, i.e., for $\theta \in [0,T) \cup (180-T, 180]$, there is no initial state $|\psi\rangle$ that makes $\{|\phi_i\rangle\}$ mutually orthogonal. □

Partition of the Coefficients into Symmetric Coefficients. Let n be the number of sensors. For $n > 3$, the coefficients $|\psi_i|^2$ ($1 \le i \le 2^n - 2$) are not "symmetric." In fact, the set of all 2^n coefficients can be partitioned into $n+1$ sets $\{S_k\}$ of symmetric coefficients as below.

$$S_k = \{|\psi_j|^2 \mid \text{Eigenvector } |j\rangle \text{ has } k\ u_+\} \quad \forall 0 \le k \le n.$$

For a partition S_l, let R_l be the number of coefficient from S_l in RHS of an equation corresponding to Equations 7-9 (omitted due to poster space constraint) and L_l be the number of coefficient from S_l in the LHS.

Theorem 2: For a given $\theta \in [T, 180-T]$ degrees, where T is defined in Theorem 1, the initial state $|\psi\rangle$ that lets the n states $\{|\phi_i\rangle\}$ mutually orthogonal is as follows.

Let S_l be the partition that minimizes the ratio R_l/L_l, where R_l and L_l are as defined above. We have,

$$L_l = |S_l| \times \frac{\lceil\frac{n}{2}\rceil}{2\lceil\frac{n}{2}\rceil - 1}$$

$$R_l = |S_l| \times \frac{\lceil\frac{n}{2}\rceil - 1}{2\lceil\frac{n}{2}\rceil - 1}$$

$$|S_l| = \binom{(n)}{(\lfloor\frac{n}{2}\rfloor)}$$

The coefficients of the optimal $|\psi\rangle$ are given by:

$$|\psi_j|^2 = \frac{1}{|S_l| - L_l\cos 2\theta - R_l} \qquad \forall\ |\psi_j|^2 \in S_l$$

$$|\psi_j|^2 = \frac{-L_l\cos 2\theta - R_l}{|S_l| - L_l\cos 2\theta - R_l} \qquad \forall\ |\psi_j|^2 \in S_0$$

$$|\psi_j|^2 = 0 \qquad \forall\ |\psi_j|^2 \notin S_l \cup S_0$$